Scuola Superiore di Catania

Corso Specialistico

a.a. 2024-2025

Metodi Geometrici in Meccanica Quantistica e Teoria dei Campi

Geometrical methods in Quantum Mechanics and Field Theory

Prerequisiti richiesti:

Analisi I e Analisi II

Obiettivi formativi:

This course aims to introduce students to the significance of topology and modern geometrical methods in physics, particularly in Quantum Mechanics and (classical) Field Theory. The initial segment of the course will cover fundamental topological and algebraic concepts. Students will then be progressively introduced to the Fundamental Group and Exact Sequences, with a focus on their applications in physics. Additionally, students will become acquainted with the language of differential forms and cohomology. The second part of the course shifts focus to more advanced topics, including Supersymmetry (SUSY) in Quantum Mechanics, Anomalies, generalized symmetries, and bosonic string theory.

Contenuti delle lezioni:

Introduction to topological and geometrical methods in physics (20 hours) Topological Space Manifolds Homotopy Groups Properties of the Fundamental Group Example of Fundamental Groups Retracs Higher Homotopy Groups Homotopy Groups and Exact Sequences Calculus on Manifolds Differential Forms Exterior Derivatives Closed and Exact Differential forms Interior Product, Exterior Derivatives and Lie Derivative Integration of Differential Forms Lie Groups and Algebras Irreducible Representation of Lie Algebras The Defining Representation and Adjoint Representation Examples Part 2 (15hours) Supersymmetry in spacetime and Quantum Mechanics Poincare' group and its odd extensions; superalgebra in 4d;consequences of supersymmetry: degeneracy of mass spectrum, energy of the vacuum, parity of even/odd states; Witten index; representations of massless and massive particles. SUSY in QM: harmonic oscillator, explicit realization of superalgebra and cancellation of vacuum energy. More general potential: argument and conditions for dynamical SUSY breaking; non-perturbative verification by explicit computation (tunneling wave-function). Anomalies in QM and examples Generalized symmetries in Field Theory Review of the Noether theorem for continuous symmetries and relations to topological defects Generalization to higher form symmetries Example of Maxwell theory: 1-form symmetries. Wilson and t' Hooft lines. Background gauge fields and a mixed anomaly Dependence on the global structure of the gauge groups, U(1), R and O(2) Maxwell. String theory and critical dimension Definition of Nambu-Goto vs Polyakov action. Quantization of the closed

string: reduction to a QM of the oscillators, plus constraints. Normal ordering and vacuum energy. Spacetime spectrum and consistency with Lorentz invariance: determination of the critical dimension.

Metodologia didattica:

Lezioni frontali

Modalità della verifica finale di apprendimento:

prova scritta e orale