Scuola Superiore di Catania

Corso specialistico

a.a. 2019-2020

Limiti di successioni di insiemi ben fondati: un varco di accesso agli iperinsiemi

Limits of sequences of well-founded sets: a gate to hyperset theory

The classical notion of *limit of sequences of sets* turns out not to be well-suited when sets possess an internal structure, as is the case, for instance, for sequences of maps or, more in general, for sequences of well-founded sets. Besides Kolmogorov's limit of sequences of sets of points in a topological space, and its generalizations, in literature no proposal for the limit of sequences of well-founded sets is currently present.

This course provides a first definition for the limit of sequences of well-founded sets and studies its properties in-depth. Its generalization to urelements encompasses the classical notion of limit of set sequences. Besides the cases in which the limit of a sequence of sets is a *hereditarily finite set*, it turns out that the limit is always a *proper hyperset*. This is a consequence of a fundamental dichotomy property, which will be established at the end of the course. In this sense, converging sequences of well-founded sets can be seen as an alternative approach to the theory of hypersets.

Tentative syllabus

- Introduction to Hilbert-style deductive systems for first-order logic.
- Introduction to well-founded set theory: axioms, ordinal numbers, von Neumann cumulative hierarchy.
- Introduction to non-well-founded set theory (hyperset theory): Anti-Foundation axiom and Strong Extensionality.
- Classical definition of the limit of set sequences.
- Kuratowski's definition of limit of set sequences in topological spaces.
- Limits of the classical set limit.
- Generalization of the notion of limit to sequences of well-founded sets with or without *urelements*.
- Co-convergency: stable-members sequences (sms), frequent-members sequences (fms), 'open-convergency' inference rule.
- Constant hereditarily finite sequences.
- Well-founded and non-well-founded sequences.
- Amalgamations and subsequences.
- Representative systems of convergent sms's.
- Strongly non-convergent sequences, thick sequences and necessarily slim sequences.

Slim sequences and the Dichotomy Theorem for Slim Sequences.